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Short Communication

# Determination of time-varying contact length, friction force, torque and forces at the bearings in a helical gear system

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**Abstract**

This paper deals with determining various time-varying parameters that are instrumental in introducing noise and vibration in a helical gear system. The most important parameter is the contact line variation, which subsequently induces friction force variation, frictional torque variation and variation in the forces at the bearings. The contact line variation will also give rise to gear mesh stiffness and damping variations. All these parameters are simulated for a defect-free and two defective cases of a helical gear system. The defective cases include one tooth missing and two teeth missing in the helical gear. The algorithm formulated in this paper is found to be simple and effective in determining the time-varying parameters.
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## 1. Introduction

Helical gears have a smoother operation than the spur gears because of a large helix angle that increases the length of the contact lines [1]. Moreover, they are also effective in resisting the axial thrust. But these functions of helical gears are also the root cause in exciting a number of time-varying parameters. A number of literature have dwelt upon the importance of the time-varying parameters excited in the operation of a helical gear system. These parameters are the contact line variation [1–6], gear mesh stiffness variation [4,5], gear mesh damping variation [7], friction force [5,8–11] and torque [8–10] variation, and bearing force variation [5,8]. In some of the articles, the coefficient of friction was also considered as a linear [9] and nonlinear [10] time-varying parameter. Martin [12] has reviewed this variation of coefficient of friction with various lubricating systems. Two review papers [13,14] have already been published to explain the effects of all the time-varying parameters on exciting noise and vibration.

Some of the important aspects of the literature are as follows. In most of the articles, the load distribution across a tooth has been assumed to be uniform [2]. Variation in the bearing forces is very large at low speed operation than at large speed operation [8]. The reason of dynamic response of a gear system is mainly due to

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**Nomenclature**

| | |
|---|---|
| $a_1$, $a_2$ | distance of the midpoint of the gear in the transverse plane from left and right side bearings, respectively |
| $b_1$ | width of gear and pinion |
| $C_g(t)$ | time-varying damping coefficient |
| $F$ | total normal force |
| $F_{ij}$ | Normal force at $i$th line and $j$th segment |
| $F_{bi}$ | forces at the bearings due to friction force $F_f$, $i = \mathrm{LT}$ or RT denote the left and right transverse force |
| $F_f$ | friction force |
| $k_0$ | tooth mesh stiffness constant |
| $K_g(t)$ | time-varying stiffness coefficient |
| $l$ | length of contact zone/pressure plane |
| $l_1$,$l_2$,$l_3$,$l_4$ | various distance shown in Fig. 2. |
| $L_f/\zeta$ | moment arm of the friction force |
| $L_i$ | The $i$th contact line, where $i = 1$, 2 and 3 |
| $L_{ij}$ | $i$th contact line and $j$th segment |
| $M_{\mathrm{LT}}$, $M_{\mathrm{RT}}$ | moments of the friction force at the left and right side of the bearings in the transverse plane |
| $N$, $n$ | number of teeth of gear and pinion, respectively |
| $p_t$ | transverse pitch |
| $p_a$ | axial pitch |
| $p_n$ | normal pitch |
| $R_a$, $r_a$ | addendum circle radius of gear and pinion, respectively |
| $R$, $r$ | pitch circle radius of gear and pinion, respectively |
| $t$ | instantaneous time |
| $T_f$ | frictional torque |
| $v$ | pitch line velocity |
| $\alpha$ | helix angle |
| $\alpha_b$ | base helix angle |
| $\phi$ | normal pressure angle |
| $\phi_t$ | transverse pressure angle |
| $\mu$ | coefficient of friction |

the variation in the friction force rather than the magnitude of the friction force [11]. There are two effects of variation in the friction force in spur gears, viz., the input and output torque no longer remains constant and a global tangential displacement is introduced that further induces a time-varying dynamic transmission error [15]. The variation of coefficient of friction has been attributed to a number of parameters, such as tooth geometry, surface hardness, axial velocity, contact pressure and misalignment [16]; contact ratio, speed ratio, transmitted load and lubricating condition [17]; and viscosity and sliding velocity [18]. Though the sliding friction is very prominent, Michlin and Myunster [19] have found that the rolling friction cannot be neglected as it causes frequent spalling near the pitch point.

## 2. Objective and procedure

This paper is devoted to determine variation in the number of contact lines in the meshing zone of a helical gear system. An example of the meshing action of a helical gear system is illustrated in Fig. 1. The pressure plane or contact zone of the gear meshing is limited by line of the intersection of the addendum circles of the gear and pinion with the tangent drawn through the two base circles. The contact lines are formed by the engaged teeth of the meshing gear and pinion. Since the width of the gear and pinion is not an integral multiple of the axial pitch, there is a continuous variation of the contact lines with the phase of engagement. The terminologies used in this paper is as given below:

$$
\begin{aligned}
l_1 &= b_1 \tan \alpha_b - l, \\
l_2 &= 3p_t - l - b_1 \tan \alpha_b, \\
l_3 &= \sqrt{r_a^2 - (r \cos \phi_t)^2} - r \sin \phi_t, \\
l_4 &= 2p_t - b_1 \tan \alpha_b, \\
l_5 &= \frac{b_1}{2} \tan \alpha_b - l_3,
\end{aligned} \tag{1}
$$

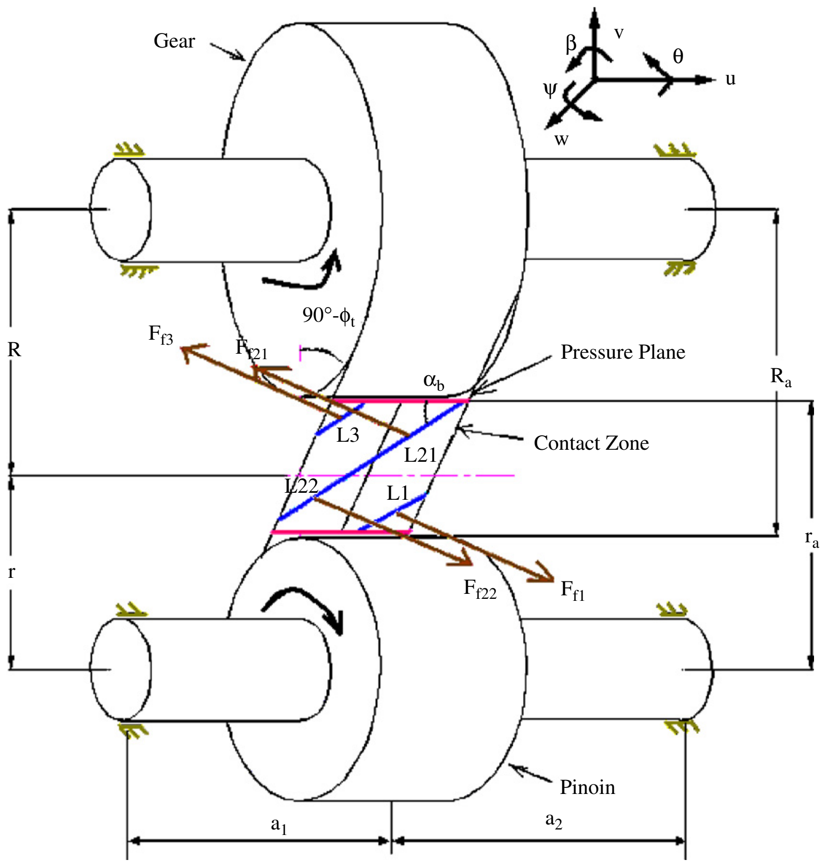


Fig. 1. Meshing of pinion and gears with three contact lines and the friction forces on the contact zone.

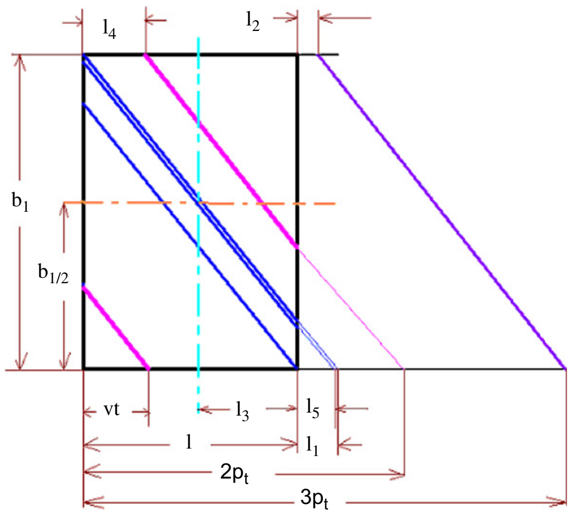


Fig. 2. Terminology used in the helical base plane.

where

$$l = \sqrt{R_a^2 - (R\cos\phi_t)^2} + \sqrt{r_a^2 - (r\cos\phi_t)^2} - (R + r)\sin\phi_t.$$

The base helix angle ($\alpha_b$) and transverse pressure angle ($\phi_t$) can be obtained from the following equations [20]:

$$\sin \alpha_b = \sin \alpha \cos \phi, \tag{2a}$$

$$\tan \phi_t = \tan \phi \sec \alpha. \tag{2b}$$

The variation in the gear mesh stiffness is determined by multiplying a stiffness constant to the contact line variation as shown in Eq. (3). This stiffness constant is found using ISO 6336 [21]. The time-varying gear mesh damping is determined by applying Eq. (4) [7]:

$$K_g(t) = k_0 L(t), \tag{3}$$

$$C_g(t) = 2\zeta \sqrt{\frac{J_p J_g K_g(t)}{(r_b^2 J_g + R_b^2 J_p)}}. \tag{4}$$

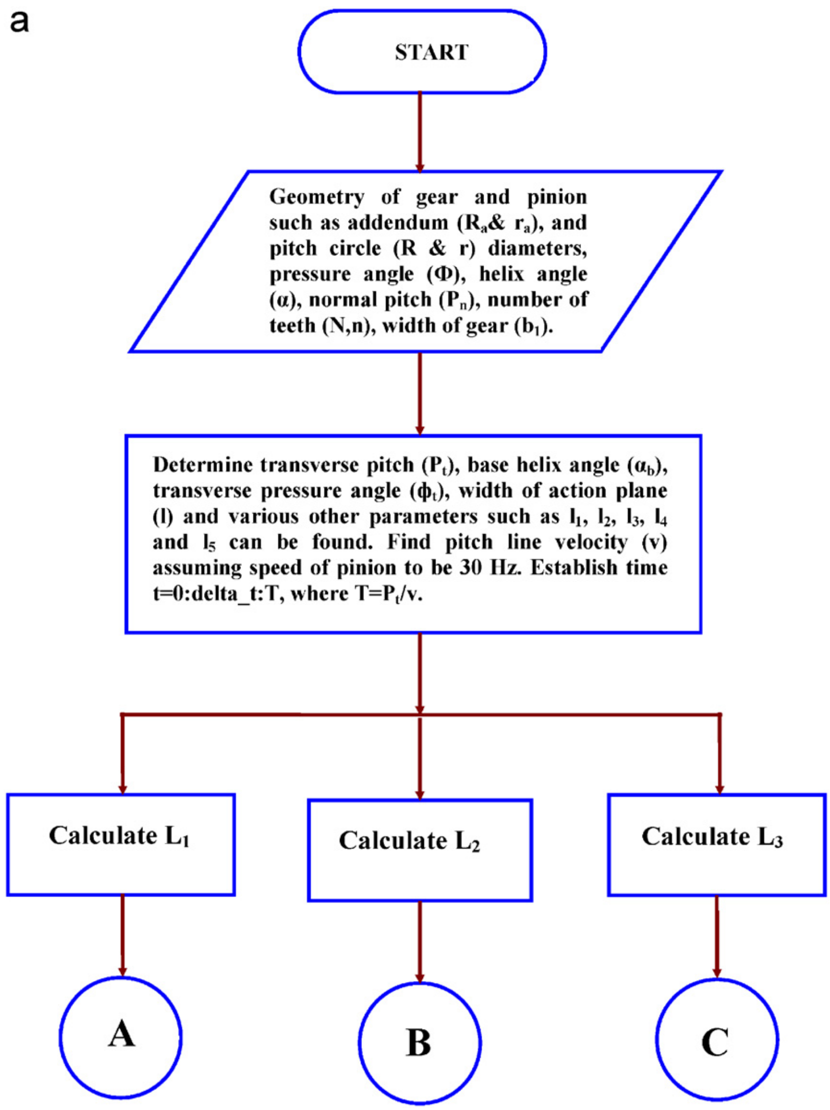


Fig. 3. Flow chart to find all the time-varying parameters of a helical gear: (a) steps taken to determine the contact line variations; (b) flow chart for first contact line ($L_1$); (c) flow chart for second contact line ($L_2$); (d) flow chart for third contact line ($L_3$); and (e) flow chart for finding total variations for one transverse pitch travel.

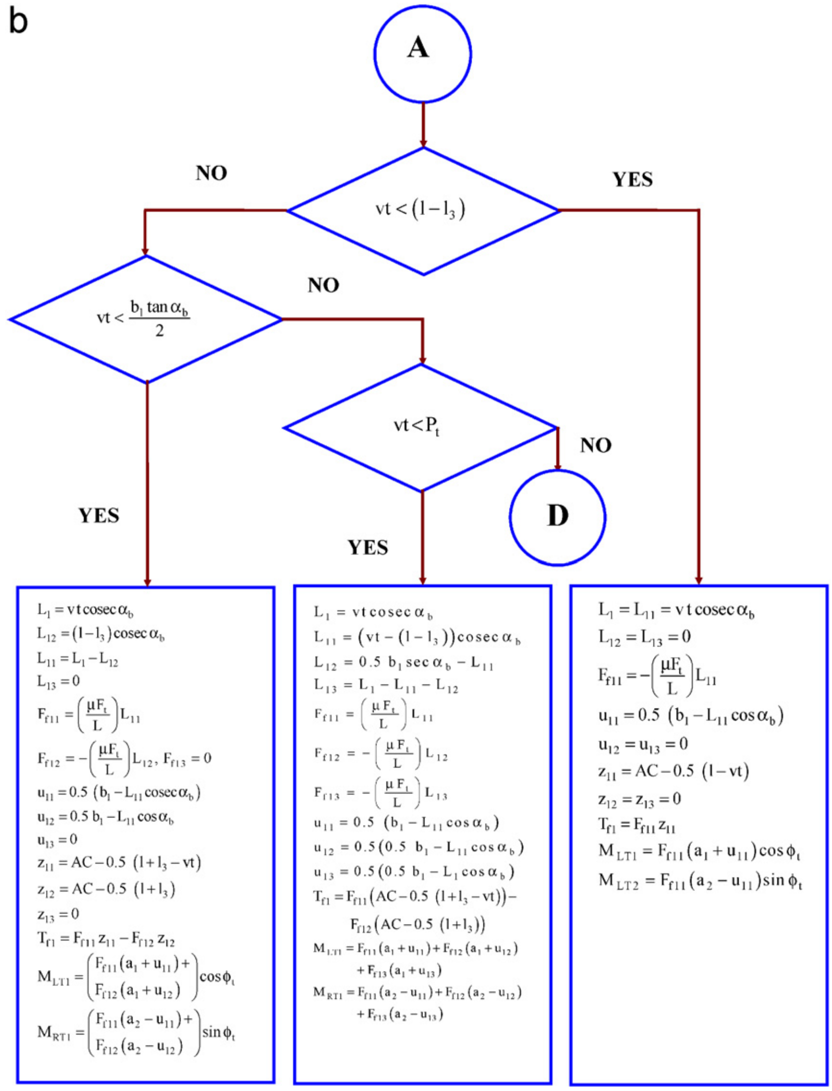


Fig. 3. (*Continued*)

To determine the friction force, each contact line is divided into two segments, one is above the pitch plane, and the other is below the pitch plane. The sign convention of the friction force is negative if the position of contact length is below the pitch plane and positive if the same is above the pitch plane. The moment arm of the friction force, which is useful for calculating frictional torque is found by taking the center point of this segment. The sign convention of torque is positive when it assists the rotation. For calculating the variation in the bearing force, the contact line segments are further segmented depending upon their position; from left or right to the midpoint of the face width of the gears. So any contact line will be divided into maximum of three segments depending upon its position with respect to the pitch plane and the line dividing the width of the gear. Hence the total force will be calculated for maximum of three segments. The moment arm is determined by the position of each force from the bearing position. Contact length, friction force and torque for a defect-free and two defective cases of helical gears are predicted and their effects are analyzed. The defects in the helical gears are one tooth removed or two consecutive teeth removed.

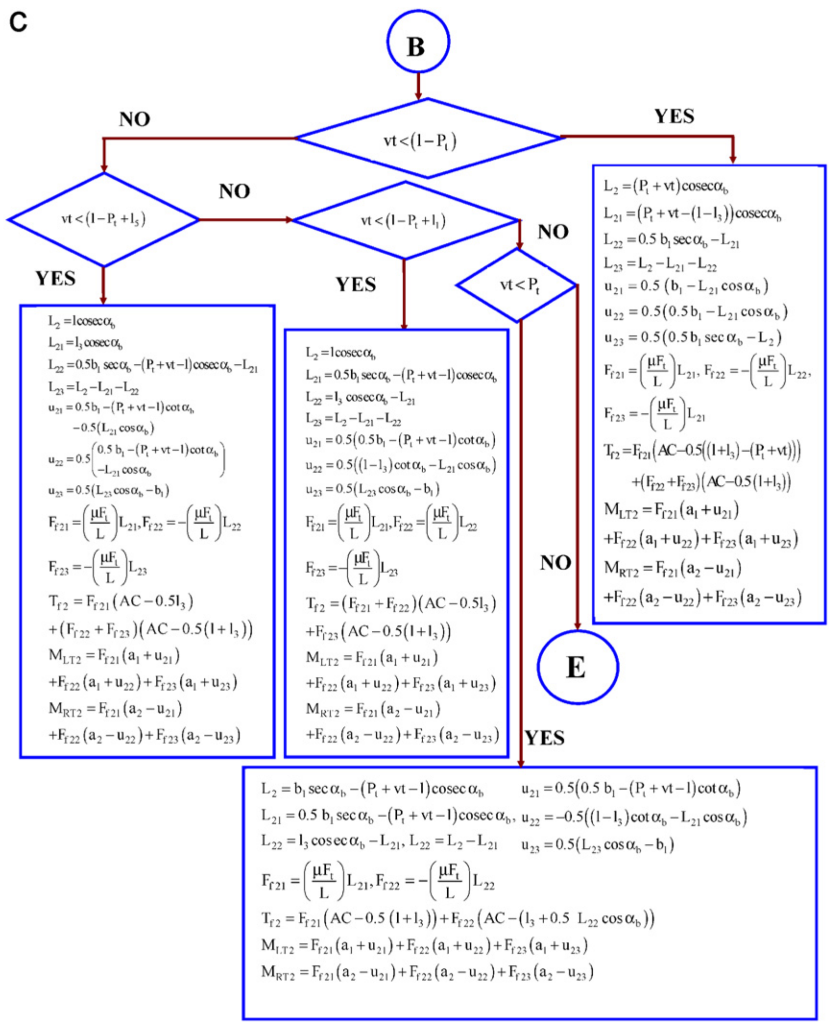


Fig. 3. (*Continued*)

Following are the assumptions taken in the formulation:

1. The contact zone travels with the velocity of pitch surface.
2. Only two degree of freedom, i.e. the rotation of gears are assumed in this paper.
3. Load distribution across the contact line is uniform.
4. Though only sliding friction is considered in this paper, the coefficient of friction is assumed to be constant with a value of 0.16 [14] considering the findings of Michlin and Myunster [19].
5. The width of the contact zone '*l*' must lie in the range of $p_t$ and $2p_t$ which is the case in most of the gears.
6. No two contact lines touch both the faces simultaneously.

## 3. Algorithm

The algorithm to determine all the time-varying parameters is illustrated in Fig. 3, where the contact lines are determined as per the maximum number of contact lines present in the system, i.e. 3, as the contact ratio of the helical gear system is 2.9. Therefore, there is a phase delay of one transverse pitch for each consecutive

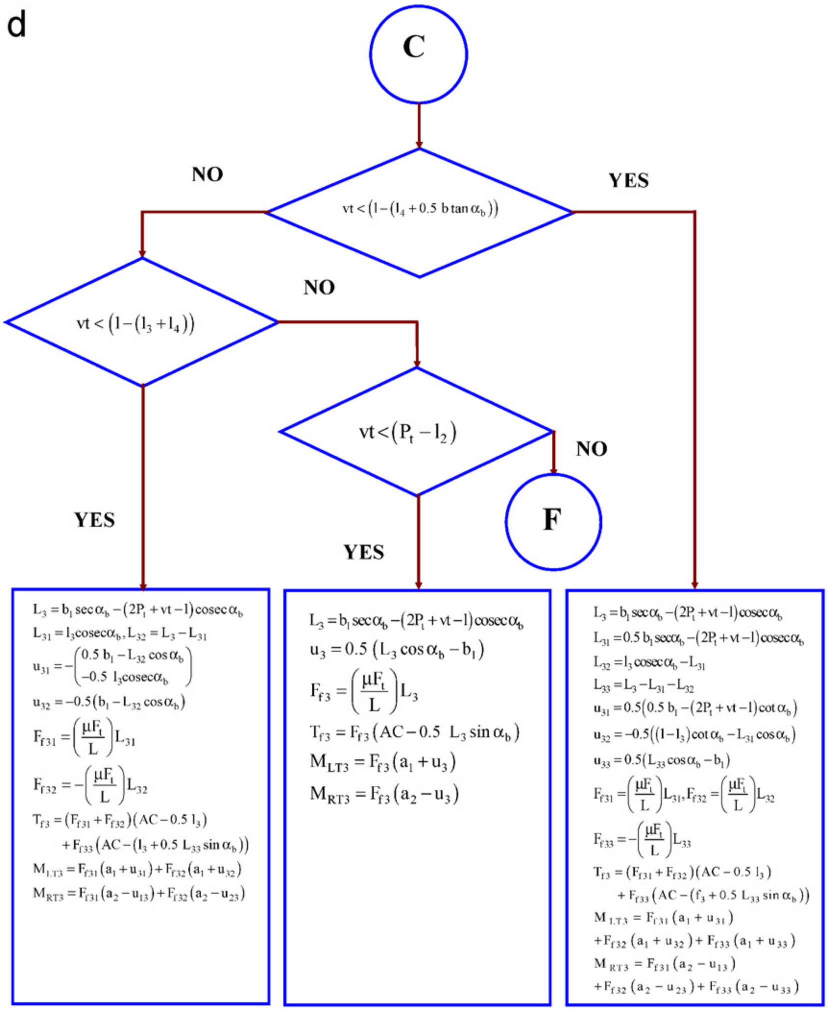


Fig. 3. (*Continued*)

contact line. The friction force, frictional torque and forces at the bearings are determined as per the individual contact line passing through the whole contact zone.

## 4. Results and discussion

The time-varying parameters such as contact lengths, friction force, frictional torque and forces at the bearings for the defect-free helical gear system are determined for a helical gear system whose specifications are provided by the manufacturer and described in Table 1. The algorithm is applied to a normal operating helical gear and defective helical gears, such as one tooth removed and two consecutive teeth removed in the pinion. The results are discussed below.

### 4.1. Normal operating gear

The contact length variation in the contact zone is a piecewise linear function as illustrated in Fig. 4a. Since any of the contact lines does not touch both the faces simultaneously, the maximum contact length per each tooth is $l \operatorname{cosec}\alpha_b$. It is to be noted that Velex and Sainsot [5] have found the maximum length of the contact line as $b \sec \alpha_b$. The distance for which the contact length becomes zero is $l_2$ as shown in Fig. 2. The limitation

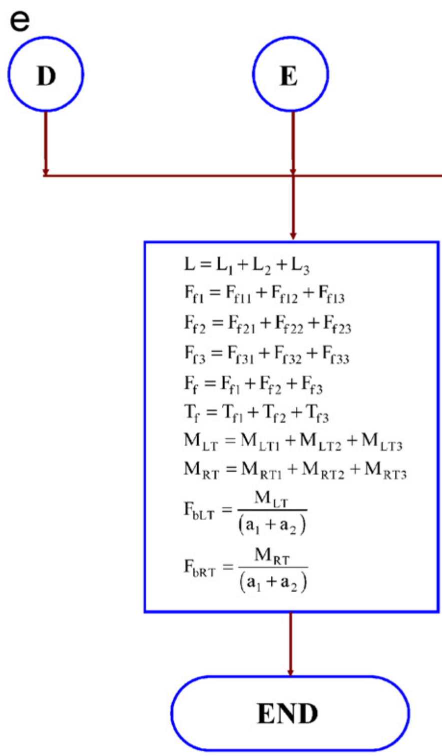


Fig. 3. (*Continued*)

of this algorithm is that it is suitable for any system with $p_t \leqslant l \leqslant 2p_t$. The variation has a longer constant contact length, which is due to the simultaneous increase of length for $L_1$ and decrease of contact length $L_3$ and constant contact length $L_2$. Fig. 4b denotes the friction force in all the three teeth for one transverse pitch travel of the pinion. It has confirmed the observation by Iida et al. [18], while investigating spur gears that the friction force is periodic but not sinusoidal. The time-varying frictional torque when three teeth move for a distance of transverse pitch is given by Fig. 4c. It can be observed that the curves are smooth, though only two numbers of segments of each contact lines such as above the pitch plane and below the pitch plane are considered in this paper contrary to a large number segments suggested in Ref. [5]. It can be observed that the sign of friction torque is negative with a mean torque of $-409.87$ N mm (Table 2), this is in accordance to the fundamental of frictional torque that it opposes the rotation.

For 21 number of teeth in the pinion, any variation is repeated 21 times to give rise to the time-varying parameter for the pinion for one rotation as illustrated in Fig. 5a for contact line variation. This variation, subsequently gives rise to time-varying mesh stiffness [5] and time-varying damping coefficient [7]. Fig. 5b and c denote the frictional torque variation and left hand transverse force at the bearings.

### 4.2. Defective gear

The algorithm is simulated for two cases of defective gears such as one tooth removed from the pinion and two teeth removed from the pinion. For the case of one tooth removed, one contact line will be missing in the contact zone, e.g. for the first transverse pitch, $L_1$ is considered as zero whereas in the next transverse pitch, $L_2$ is considered as zero. For the last transverse pitch, the $L_3$ becomes zero. All the parameters are evaluated and illustrated in Fig. 6. A sudden fluctuation in all the parameters is observed in the figure.

In case of two teeth removed, two contact lines will be missing and hence five transverse pitch will be affected. In the first transverse pitch travel, only $L_1$ will be zero. In the next transverse pitch, $L_1$ and $L_2$ will be zero. Then $L_2$ and $L_3$ will be zero and in the last transverse pitch travel, $L_3$ will be zero. The second transverse

Table 1
Specification of the helical gear pair

| Sl. no. | Item | Pinion | Gear |
|---|---|---|---|
| 1 | Addendum circle diameter (mm) | 57.2262 | 77.5462 |
| 2 | Pitch circle diameter (mm) | 53.34 | 73.66 |
| 3 | No. of teeth | 21 | 29 |
| 4 | Rotational speed (Hz) | 30 | 21.72 |
| 5 | Base width ($b$) (mm) | 14.986 | |
| 6 | Helix angle ($\alpha$) (deg) | 40 | |
| 7 | Pressure angle ($\phi$) (deg) | 14.5 | |
| 8 | Transverse contact ratio | 1.4146 | |
| 9 | Overlap contact ratio | 1.5758 | |
| 10 | Total contact ratio | 2.9905 | |

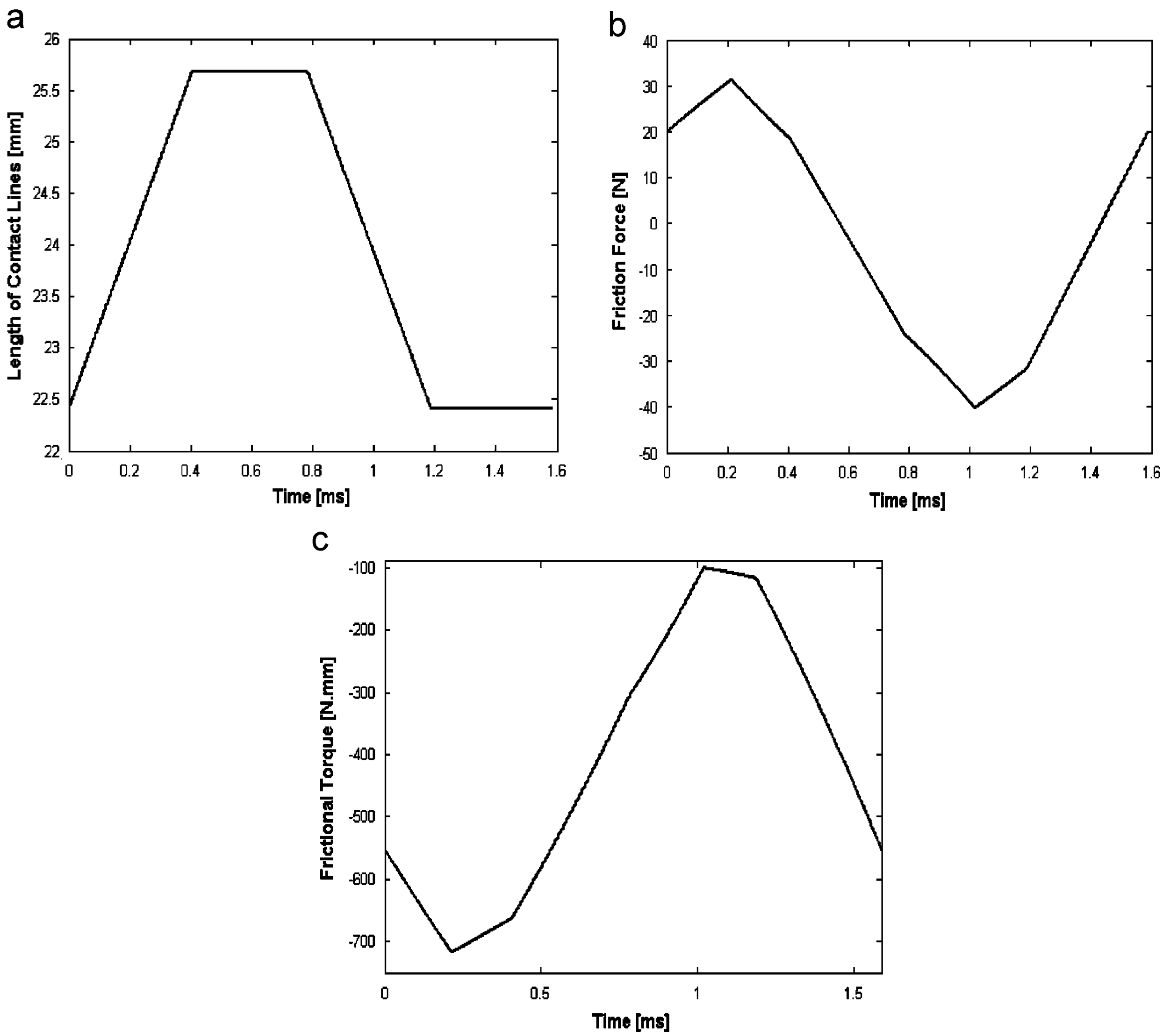


Fig. 4. Time-varying parameters in the pinion of the helical gear system for one transverse pitch travel: (a) length of contact lines; (b) friction force; and (c) frictional torque.

Table 2
Statistical parameters of the variations of length of contact line, friction force and frictional torque for one transverse pitch travel

| Sl. no. | Item | Maximum | Minimum | Mean |
|---|---|---|---|---|
| 1 | Contact length (mm) (Fig. 4b) | 25.68 | 22.41 | 24.03 |
| 2 | Friction force (N) (Fig. 6b) | 31.33 | −40.12 | −4.17 |
| 3 | Frictional torque (N mm) (Fig. 8b) | −100.78 | −717.15 | −409.87 |

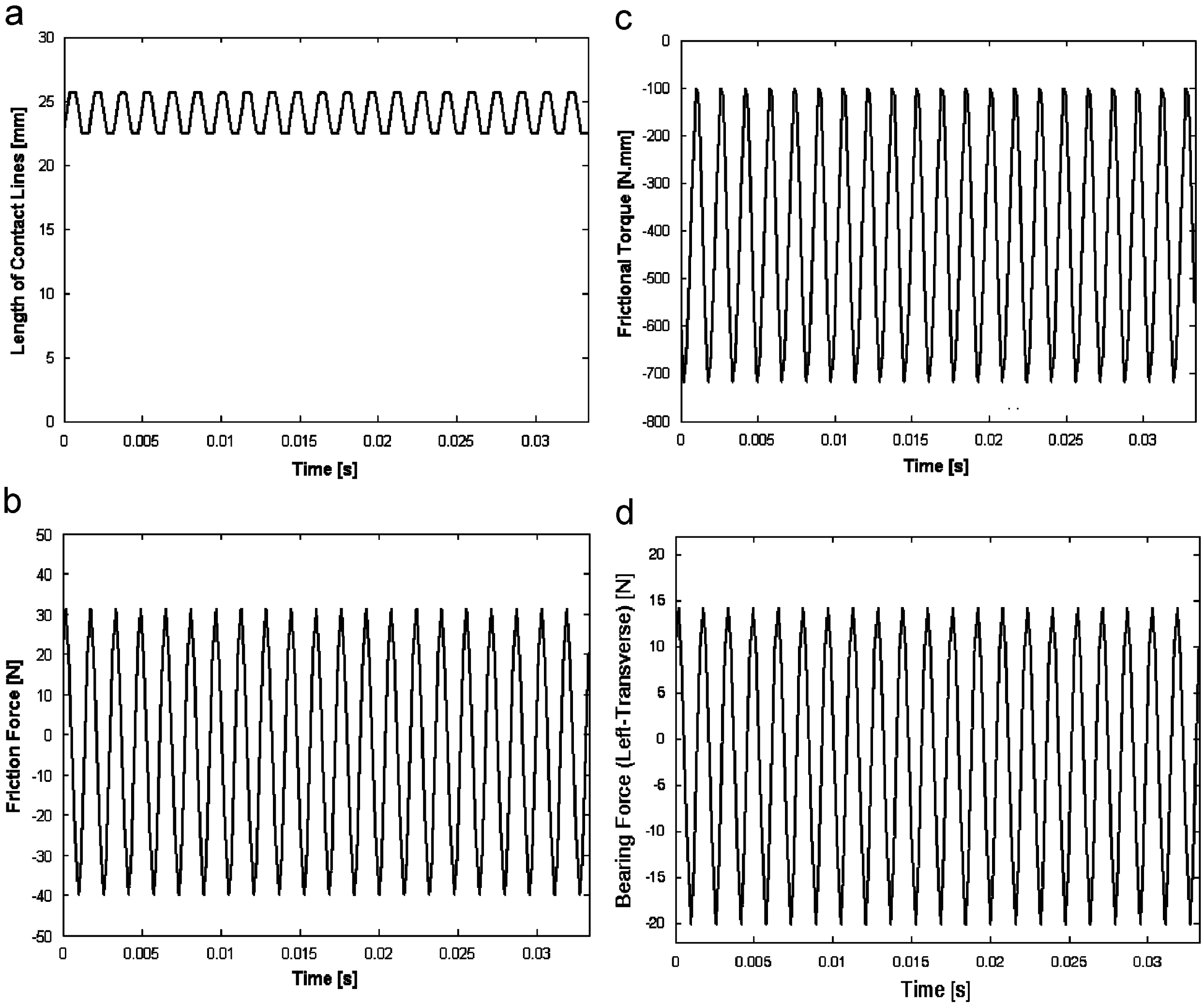


Fig. 5. Time-variation of following parameters for one rotation of the second counter gear: (a) contact lines; (b) friction force; (c) frictional torque; and (d) left-side bearing force in transverse direction.

pitch at which $L_1$ and $L_2$ are zero, the total contact lines will be zero for $l_2$ (Fig. 2) distance as the $L_3$ is also zero. Therefore, the friction force, frictional torque and forces at the bearings are also zero. Fig. 7 illustrates the parameters for the case of two teeth removed, where it can be observed that there is a contact loss. The FFT analysis of the friction force for the defect-free and defective cases is illustrated in Fig. 8 in order to explain the effect of the defects on these parameters. It is observed that energy smears into low frequency with increase in tooth breakage.

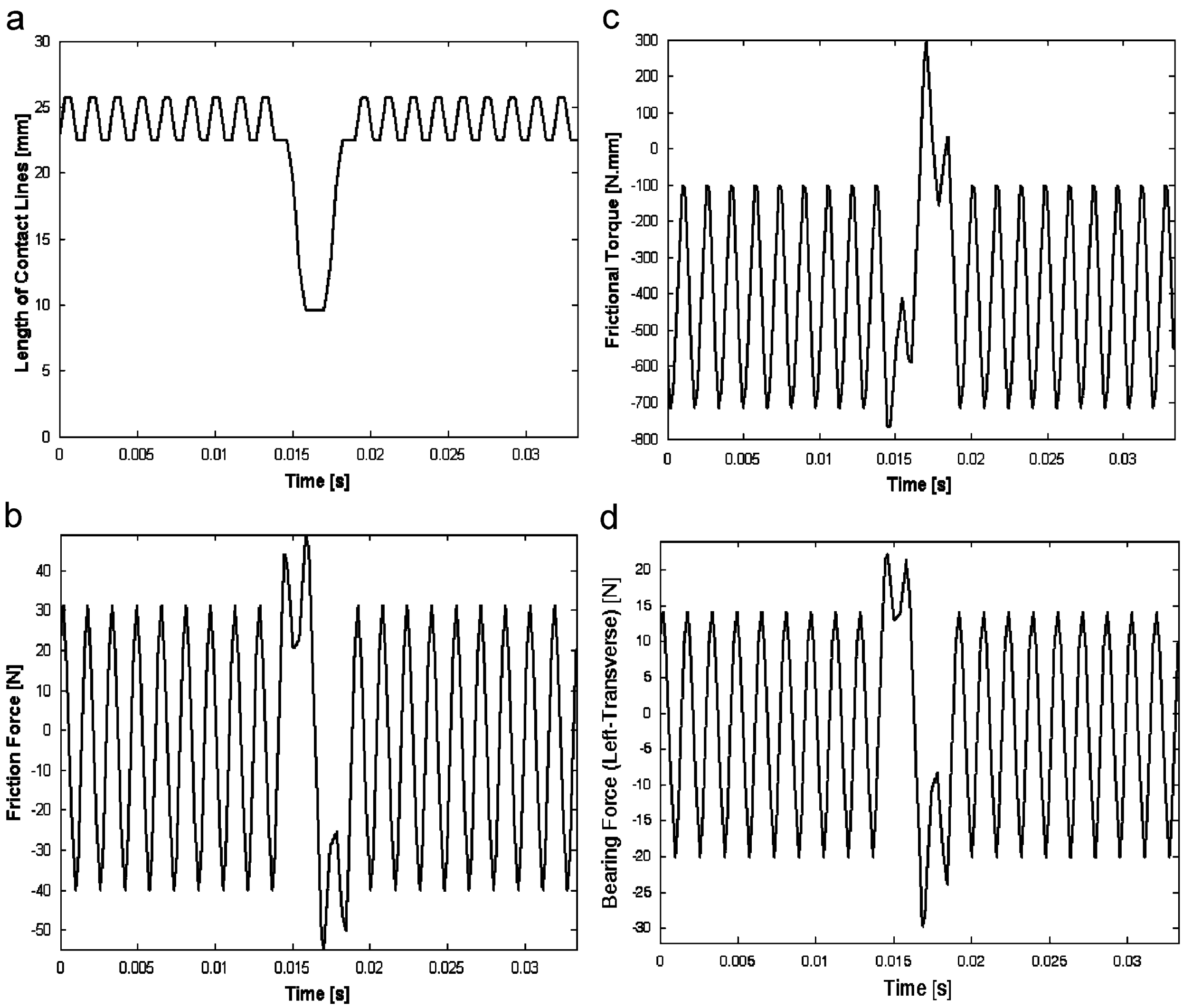


Fig. 6. (a) Contact line variation; (b) friction force; (c) frictional torque; and (d) bearing force on left-side of the pinion for the case of one tooth missing.

## 5. Conclusion

The paper is devoted to determine various time-varying parameters using a simple algorithm. These parameters are contact line variation, friction force, frictional torque and forces at the bearings. The variation in the contact lines is determined by the geometry condition of the helical gear. This variation further gives rise to variation in all other above-mentioned variation including gear mesh stiffness and damping variation. The friction force and the frictional torque are determined by dividing the contact line into two segments, whereas the forces at the bearings are determined by dividing any contact line into maximum of three segments. The algorithm is applied to a defect-free and defective helical gear system where the defective conditions are one tooth removed and two teeth removed from the pinion. A contact loss is observed for the case of two teeth removed. It is also observed that tooth removal causes a low frequency excitation in the parameter.

The parameters can be arranged as Fourier series functions that can be used to calculate the vibration response of any helical gear. Such types of functions for gear mesh stiffness and damping, and frictional torque have already been used by Vaishya and Singh [10] in order to determine the torsional vibration

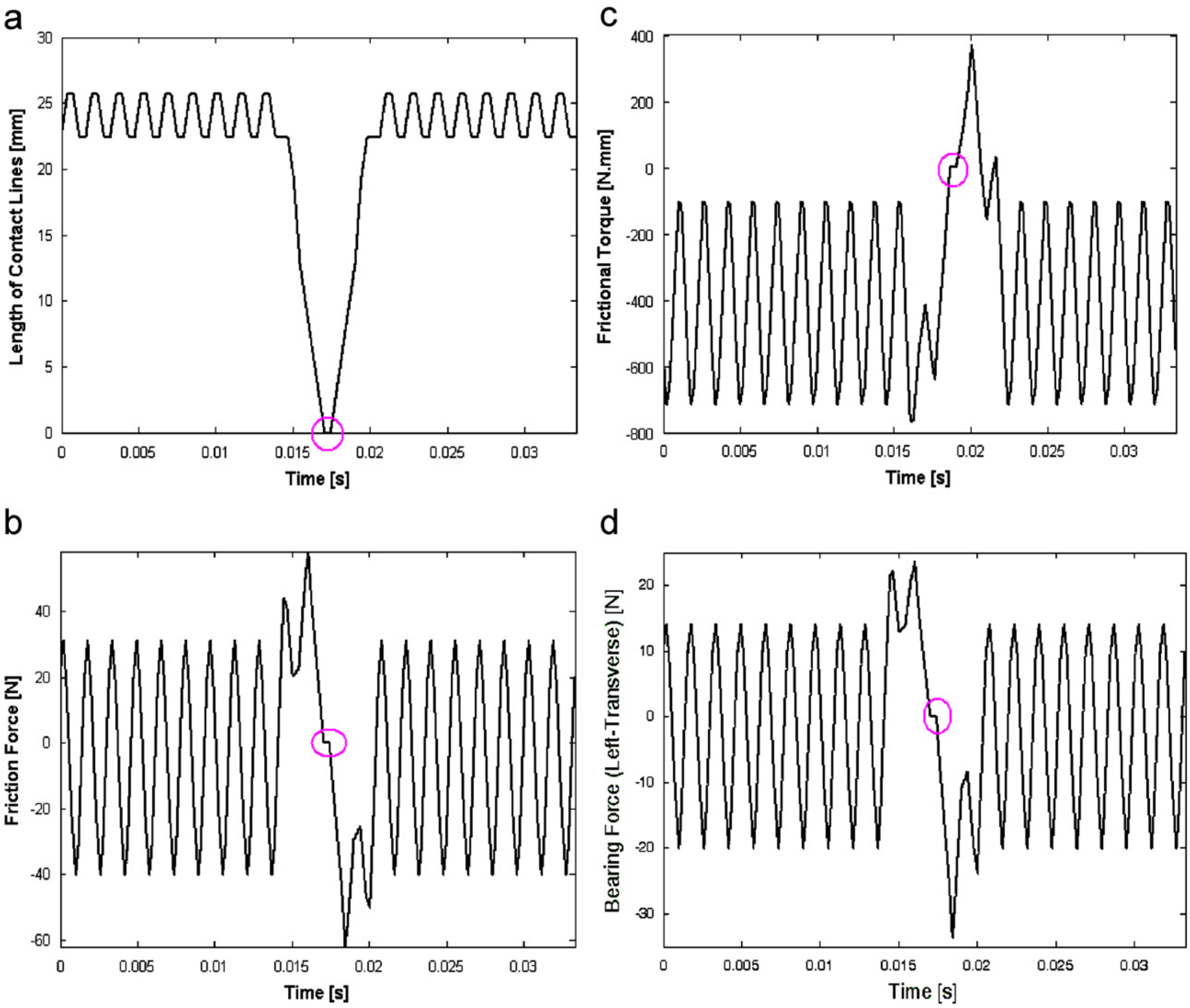


Fig. 7. (a) Contact line variation; (b) friction force; (c) frictional torque; and (d) bearing force on left-side of the pinion for two teeth missing condition.

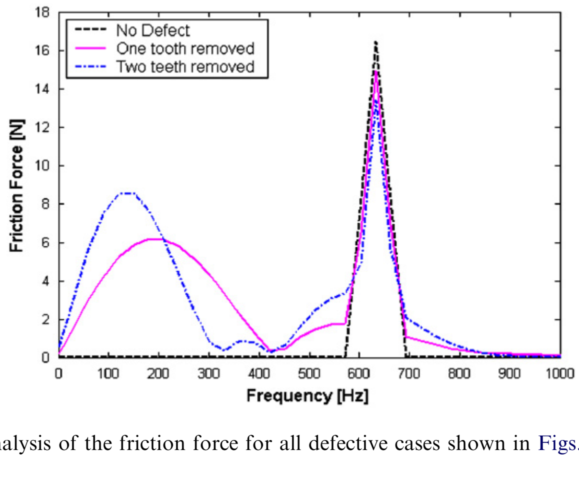


Fig. 8. FFT analysis of the friction force for all defective cases shown in Figs. 5b, 6b and 7b.

response of a spur gear. The forces at the bearings will facilitate in determining the coupled torsional and lateral vibration response of the helical gear system.

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